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| 14. ABSTRACT<br>New stability analysis tools and high performance control strategies were developed for nonlinear systems. Robust, high performance anti-windup algorithms were produced. New, comprehensive nonlinear stability analysis tools were discovered and used to motivate new control algorithms. A robust stability theory for discontinuous discrete-time systems was developed. New results on output feedback for nonlinear systems were described. A comprehensive multiple time scale stability theory for systems with disturbances was codified. A theory for extremum seeking in dynamical systems was developed and applied to industrial engine calibration problems. |                             |                              |                               |  |  |
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**FINAL REPORT**

**APPLIED NONLINEAR CONTROL DESIGN**

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## Summary

The objective of this research was to develop high performance control strategies for nonlinear systems, solving nonlinear control problems that are important in industrial applications, especially in the aerospace industry. In particular our goals, which we reached successfully, were 1) to further develop the robust, high performance anti-windup synthesis theory initiated and demonstrated under a previous AFOSR sponsored project; 2) cooperate with industry to apply these anti-windup techniques to modern aerospace applications and to vibration isolation problems that assist high precision manufacturing industries; 3) to provide new, comprehensive stability analysis tools for nonlinear systems that inspire new control concepts; 4) provide insight into principles and algorithms for a) robust, discontinuous sampled-data nonlinear control design, b) nonlinear output feedback control design, c) time-varying control design, and d) multiple time scale control design. The results of our work are described in fifteen journal publications, two book chapters, thirty-seven conference papers and eleven additional journal papers currently under review.

## Research Publications

The research supported by this grant resulted in 15 journal papers published or in press and an additional 11 journal papers submitted. It resulted in 2 book chapter and 37 referred conference papers, published or to appear. The published papers are listed below.

### Journal papers and book chapters

1. "A Benchmark Example for Anti-Windup Synthesis in Active Vibration Isolation Tasks and an L2 Anti-Windup Solution," L. Zaccarian and A.R. Teel, *European Journal of Control*, vol. 6, no. 5, pp. 405-420, 2000.
2. "Robust Nonlinear Control of Feedforward Systems with Unmodeled Dynamics," M. Arcak, A.R. Teel, and P. Kokotovic, *Automatica*, vol. 37, (no. 2), p. 265-272, February 2001.
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5. "Relaxed Persistency of Excitation for Uniform Asymptotic Stability," E. Panteley, A. Loria and A.R. Teel, *IEEE Transactions on Automatic Control*, 46 (12), pp. 1874-1886, December 2001.
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9. "Changing supply functions in input to state stable systems: the discrete-time case," D. Nesic and A. R. Teel, *IEEE Transactions on Automatic Control*, June 2001, vol. 45, no. 6, pp. 960-962, 2001.
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12. "Open- and closed-loop dissipation inequalities under sampling and controller emulation," D.S. Laila, D. Nesic and A.R. Teel, *European Journal. of Control*, 8 (2), 2002, pp. 109-125, 2002.
13. "A common framework for anti-windup, bumpless transfer and reliable designs," L. Zaccarian and A.R. Teel, *Automatica*, October, 2002, vol. 38, no. 10. 2002.
14. "Input-to-state stability for a class of Lurie systems," M. Arcak and A.R. Teel, *Automatica*, November, 2002, vol. 38, no. 11, 2002.
15. "Asymptotic controllability and observability imply semiglobal practical asymptotic stabilization by sampled-data output feedback, H. Shim and A.R. Teel, *Automatica*, March, 2003.
16. "Nonlinear observer design via passivation of error dynamics", H. Shim, J.H. Seo and A.R. Teel, *Automatica*, May, 2003.
17. "On finite gain  $L_p$  stability of nonlinear sampled-data systems," L. Zaccarian, A.R. Teel, and D. Nesic, *Systems and Control Letters*, to appear, 2003.

#### Conference papers

1. "Uniform Asymptotic Controllability to a Set Implies Locally Lipschitz Control-Lyapunov Function," C.M. Kellett and A.R. Teel, *Proc. 39<sup>th</sup> IEEE Conference on Decision and Control*, pp. 3994-3999, Sydney, Australia, December 2000.
2. "UGAS of Nonlinear Time-Varying Systems: a  $\delta$ -persistence of Excitation Approach," A. Loria, E. Panteley, and A.R. Teel, *Proc. 39<sup>th</sup> IEEE Conference on Decision and Control*, pp. 3489-3494, Sydney, Australia, December 2000.
3. "Lyapunov Methods in Nonsmooth Optimization, Part 1: Quasi-Newton Algorithms for Lipschitz, Regular Functions," A.R. Teel, *Proc. 39<sup>th</sup> IEEE Conference on Decision and Control*, pp. 112-117, Sydney, Australia. 2000.
4. "Lyapunov Methods in Nonsmooth Optimization, Part II: Persistently Exciting Finite Differences," A.R. Teel, *Proc. 39<sup>th</sup> IEEE Conference on Decision and Control*, pp. 118-123, Sydney, Australia, December 2000.
5. "Set Stabilization of Sampled-Data Non-linear Differential Inclusions Via their Approximate Discrete-Time Models," D. Nesic and A.R. Teel, *Proc. 39<sup>th</sup> IEEE Conference on Decision and Control*, pp. 2112-2117, Sydney, Australia, December 2000.
6. "Averaging with Respect to Arbitrary Closed Sets: Closeness of Solutions for Systems with Disturbances," A.R. Teel, D. Nesic, and L. Moreau, *Proc. 39<sup>th</sup> IEEE Conference on Decision and Control*, pp. 4361-4366, Sydney, Australia, December 2000.

7. "On Preservation of Dissipation Inequalities Under Sampling," D. Nesic, D.S. Laila, and A.R. Teel, , *Proc. 39<sup>th</sup> IEEE Conference on Decision and Control*, pp. 2472-2477, Sydney, Australia, December 2000.
8. "Enhanced Performance Global Stabilization of Linear Systems with Saturating Actuators," R. Reginatto, A.R. Teel, and E.R. DePeiri , *Proc. XIII Brazilian Conference on Automatica*, pp. 949-954, Florianopolis, SC, Brazil, September 2000.
9. "Performance enhancement and robustness for linear systems with saturating actuators," R. Reginatto, A.R. Teel and E.R. De Pieri, *Proceedings of the 3<sup>rd</sup> IFAC Symposium on Robust Control Design (ROCOND 2000)*, Prague, Czech Republic, June 2000, pp. 427—432, 2000.
10. "Linear Matrix Inequalities for full and reduced order anti-windup synthesis," G. Grimm, I. Postlethwaite, A.R. Teel, M.C. Turner, L. Zaccarian, *Proceedings of the American Control Conference*, Arlington, VA, June 2001, pp. 4134—4139, 2001.
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12. "A trajectory based approach for robustness of input-to-state stability," L. Moreau, A.R. Teel, D. Nesic, *Proceedings of the American Control Conference*, Arlington, VA June 2001, pp. 3570—3575, 2001.
13. "Solving smooth and nonsmooth multivariable extremum seeking problems by the methods of nonlinear programming," A.R. Teel and D. Popovic, *Proceedings of the American Control Conference*, Arlington, VA, June 2001, pp. 2394—2399. 2001.
14. "Nonlinear L2 anti-windup design: an LMI-based approach," L. Zaccarian and A.R. Teel, *Proceedings of the IFAC Nonlinear Control Systems Design Symposium (NOLCOS)*, St. Petersburg, Russia, July 2001. CD-ROM.
15. "Further results on the nonlinear separation principle: the general 'asymptotically controllable' case," H. Shim and A.R. Teel, *Proceedings of the IFAC Nonlinear Control Systems Design Symposium (NOLCOS)*, St. Petersburg, Russia, July 2001. CD-ROM.
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18. "A note on the robustness of input-to-state stability," A.R. Teel, L. Moreau and D. Nesic, *40<sup>th</sup> IEEE Conference on Decision and Control*, Dec. 2001, pp. 875 –880.
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21. "Trading the stability of finite zeros for global stabilization of nonlinear cascade systems," R. Sepulchre, M. Arcak and A.R. Teel, *40<sup>th</sup> IEEE Conference on Decision and Control*, Dec. 2001, pp. 3025—3030.
22. "Backstepping on the Euler approximate model for stabilization of sampled-data nonlinear systems," Nesic, D. and A.R. Teel, *Proceedings of the 40th IEEE Conference on Decision and Control*, Dec. 2001, pp. 1737 –1742.
23. "A backstepping approach to control of active suspensions," Karlsson, N.; Teel, A.; Hrovat, D, *Proc. of 40th IEEE Conf. on Decision and Control*, 2001, pp. 4170 –4175.
24. "Control of nonlinear sampled-data systems with fixed sampling periods via their approximate discrete-time models," D. Nesic and A.R. Teel, *Information, Decision and Control Conference*, Adelaide, SA, Australia, 11-13 Feb. p.151-6., 2002.
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26. "Input-to-state stability and boundedness in Lurie systems," M. Arcak and A.R. Teel, *American Control Conference*, Anchorage, AL, May 2002, pp. 62-67.
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30. "Stabilization of sets parameterized by a single variable: application to ship maneuvering," R. Skjetne, A.R. Teel and P. Kokotovic, *Proceedings of the International Symposium on Mathematical Theory of Networks and Systems*, August, 2002.
31. "Input-to-state stability of pulse width modulated control systems," A.R. Teel, L. Moreau and D. Nesic, *Proceedings of the International Symposium on Mathematical Theory of Networks and Systems*, August, 2002.
32. G. Grimm, A.R. Teel and L. Zaccarian, "Results on an LMI approach to external anti-windup augmentation", *41<sup>st</sup> IEEE Conference on Decision and Control*, Dec. 2002, pp. 299—304.
33. G. Grimm, A.R. Teel and L. Zaccarian, "Results on robust LMI-based linear anti-windup design: optimizing the unconstrained response recovery", *41<sup>st</sup> IEEE Conference on Decision and Control*, Dec. 2002, pp. 293—298.

34. R. Skjetne, A.R. Teel and P.V. Kokotovic, "Nonlinear maneuvering with gradient optimization", *41<sup>st</sup> IEEE Conference on Decision and Control*, Dec. 2002, pp. 3926—3931.
35. Loria, E. Panteley, D. Popovic and A.R. Teel, "An extension of Matrosov's theorem with application to stabilization of nonholonomic control systems", *41<sup>st</sup> IEEE Conference on Decision and Control*, Dec. 2002, pp. 1528—1533.
36. Loria, E. Panteley, D. Popovic and A.R. Teel, " $\delta$ -Persistency of excitation: a necessary and sufficient condition for uniform attractivity", *41<sup>st</sup> IEEE Conference on Decision and Control*, Dec. 2002, pp. 3506—3511.
37. Kellett and A.R. Teel, "On robustness of stability and Lyapunov functions for discontinuous difference equations", *41<sup>st</sup> IEEE Conf on Decision and Control*, Dec. 2002, pp. 4282-4287.

## Research Accomplishments

### Anti-windup synthesis

The anti-windup problem, which is prevalent in industrial control settings, is to synthesize modifications to a predetermined linear controller that activate only upon the detection of input saturation and that minimize the performance degradation with respect to the response that would occur in the absence of saturation. This problem has been around for as long as electronic control systems have been around. For many decades, the solutions to this problem were ad-hoc in nature. In the mid to late 1990s, systematic, high performance solutions began to appear, some produced by the principal investigator under previous AFOSR support. Under the current grant, we extended our results in several directions.

*LMI-based synthesis:* In our supported work we succeeded in casting the synthesis of L2 optimal linear anti-windup compensators driven by the difference between the plant's saturated input and the corresponding unsaturated input as a problem of producing a solution to a certain set of matrix inequalities. When the anti-windup compensator's state dimension is equal to zero or to the size of the state of the plant, these matrix inequalities are linear in the parameters needed to synthesize a controller. Hence checking feasibility reduces to a convex optimization problem for which many efficient numerical algorithms exist. One of the most interesting features of the matrix inequalities is that they have a system-theoretic interpretation. In particular, the matrix inequalities correspond to the bounded real lemma for the open-loop plant, the bounded real lemma for the closed-loop plant and a coupling condition on the matrices that satisfy the bounded real lemma. In general, the coupling involves a nonconvex rank condition. However, in the special cases mentioned above, this constraint disappears. It turns out that, for a large enough L2 gain, the anti-windup synthesis matrix inequalities are feasible when the dimension of the anti-windup compensation is equal to the dimension of the plant. This is not the case for static anti-windup, in general. In that setting, a quasi-common quadratic Lyapunov function for the open and closed-loop systems must exist for the matrix inequalities to be feasible for a large enough L2 gain. We investigated the performance of anti-windup controllers synthesized using this technique on several academic and experimental systems. We followed this work with an LMI-based synthesis for anti-windup compensators with guaranteed robustness margins and also for the common external anti-windup problem in which the original controller may be augmented only at its input and at its output. In both cases, we experimented with a variety of performance metrics as part of our program to fully understand the best measure of successful anti-windup performance.

*Receding horizon optimal control-based synthesis:* As a novel contribution in the anti-windup arena, we showed that receding horizon optimal control, which is very effective at controlling constrained systems, can be readily adapted to the anti-windup problem. (It is not a solution in its raw form because its formulation does not include allowing an arbitrary small signal controller.) We showed how receding horizon optimal control for linear systems with input constraints, especially when implemented in its analytic

piecewise affine form, is extremely effective at providing high performance nonlinear anti-windup augmentation for arbitrary linear controllers.

*Euler-Lagrange systems:* While anti-windup is usually conceived for linear control systems with input saturation, we showed under a previous AFOSR grant that it can also be posed and solved for more general nonlinear feedback loops. During the current project, we applied our earlier ideas, with modifications aimed at improved performance, to Euler-Lagrange systems including robotic manipulators such as the SCARA and PUMA.

*Bumpless transfer and reliable control:* The problems of bumpless transfer and reliable control, both commonly encountered in industry, involve switching from one controller to another, either to change the response of the closed-loop system or because of the failure of some redundant sensor or actuator. We showed how our recent accomplishments in anti-windup synthesis can be used in bumpless transfer and reliable control problems to assign, by means of an extra control action, a prescribed transient behavior immediately after switching to a new controller. These results will make the action of switching between controllers less traumatic and more systematic.

### Stability Theory

*Discontinuous, discrete-time systems:* As control engineers continue to tackle nonstandard control problems and complicated constrained control problems, the need to rely on discontinuous control algorithms increases. Under this grant, we demonstrated an example that adds caution to this trend. The example shows that there are discontinuous, discrete-time systems for which the origin is uniformly globally asymptotically stable but for which this stability is not robust. In particular, asymptotically vanishing additive perturbations to the right-hand side completely destroy convergence toward the origin. This example suggests the need for a more comprehensive robust stability theory for discontinuous, discrete-time systems. In our work, we showed that asymptotic stability for an equilibrium of a discontinuous, discrete-time system is robust if and only if a regularized version of the difference equation (similar to what Filippov and Krasovskii have suggested for discontinuous differential equations) has the same asymptotic stability property. Moreover, there exists a smooth Lyapunov function that demonstrates asymptotic stability if and only if the asymptotic stability is robust. These results are destined to provide new insights on the robustness of receding horizon control algorithms with constraints and hybrid nonlinear control algorithms.

*Sampled-data synthesis based on approximate discrete-time models:* Most modern, high-performance control systems are implemented digitally, using a sampling and hold device. For this reason, sometimes it is most reasonable to design the control law for a discrete-time model of the process. Unfortunately, except for some special cases, one cannot expect to obtain an exact discrete-time model of a nonlinear process. Instead, a family of approximate models must suffice. Yet, when the discrete-time models are approximate, stabilization of each member of the approximate family does not guarantee stabilization of the exact discrete-time model. Extra conditions must be satisfied. In our

work, we elucidated what these conditions are for the problem of asymptotic stabilization of sets and discrete-time models that correspond to differential inclusions, which can model discontinuous systems and systems under the effect of worst case disturbances. We also established the extent to which dissipation results are preserved under sampling. These results will become increasingly significant as more high-performance digital control architectures make their way into industrial applications.

*Time-varying nonlinear systems:* While the Lyapunov function is the biggest workhorse for establishing asymptotic stability in nonlinear systems, additional tools, like LaSalle's invariance principle for periodic or time-invariant systems and Matrosov's theorem for general time-varying systems, often must also be introduced to draw conclusions. Matrosov's theorem, which is not as well known as LaSalle's invariance principle, suggests finding an auxiliary function that has a nonzero derivative along solutions where the Lyapunov function has a zero derivative in order to establish uniform asymptotic stability. It has been used as an analysis tool in robot tracking control problems for example. In our work, we developed an extension of Matrosov's theorem that gives it the flexibility to rival that found in LaSalle's invariance principle. In our extension, multiple auxiliary functions can be used iteratively to address regions where the Lyapunov function's derivative is zero. We have used this new theorem to provide new necessary and sufficient conditions for asymptotic stability in model reference adaptive control and in identification problems for robotic manipulators.

*Stabilization of (not necessarily compact) sets:* For stabilization of noncompact sets, we extended earlier work by Sontag and others, establishing the existence of locally Lipschitz control-Lyapunov functions under the assumption of asymptotic controllability to a (not necessarily compact) set. Following the lead of other researchers, we also developed a sampled-data proximal aiming control strategy using this control-Lyapunov function to achieve (semiglobal practical) asymptotic stabilization of noncompact sets. This result makes clear that asymptotic stabilization is possible whenever asymptotic controllability holds, and thus promotes the search for efficient control laws for a wide range of nonlinear problems. For analysis tools for asymptotic stability of noncompact sets, we provided new integral conditions for asymptotic stability for differential inclusions and have used these results to extend Matrosov's theorem to this setting.

*A unified framework for input-to-state stability in systems with two time scales:* The search for high performance nonlinear control algorithms is directed by the analysis tools that are available to certify stability, robustness and performance. We have produced a novel, unified framework for establishing input-to-state stability in nonlinear systems that exhibit multiple time scales. This framework and the resulting analysis tools produce results that cover classical averaging results, singular perturbation results and results for systems with slowly varying parameters. They also address stability for weakly nonlinear oscillators, two time scale averaging common in adaptive control and identification, partial averaging, and averaging in pulse-width modulated control systems. More significantly, the framework produces extensions to the cases where the system has exogenous disturbances and the system's attractor is complex. The framework can be viewed as a generalization of the classical singular perturbation theory where the fast,

boundary layer system contains an asymptotically stable equilibrium manifold on which a reduced system is studied to derive stability properties for the original system. In order to address various multiple time scale problems in a unified framework, we allow the asymptotic behavior of the boundary layer system to be complex and assume that it generates an average for the derivative of the slow state variables. Our main results are that if the boundary layer and average systems are input-to-state stable (ISS) in a generalized sense then the ISS bounds also hold for the actual system with an offset that converges to zero with the parameter that characterizes the separation of time scales. Because of the general notion of ISS used, it is possible to make stability statements for systems whose boundary layer system has, for example, an asymptotically stable periodic orbit parameterized by the slow state variables or an unstable but recurrent equilibrium manifold on which the averaged system is ISS. This generalized framework provides rigor for certain multiple time scale control ideas already used in practice and provides extra insight to spawn new control ideas based on multiple time scales. We used it to produce new stability guarantees for pulse-width modulated control systems with disturbances. In another application, we showed how some maneuvering control algorithms developed by our colleagues can be tuned to produce near global asymptotic stability of a maneuvering profile without sacrificing global asymptotic convergence to the maneuvering profile. Our two time scale framework, which is general enough to apply to the maneuvering problem, suggests the tuning naturally.

*Nonlinear observability and output feedback stabilization:* As mentioned above, we established that asymptotic controllability to a set implies semiglobal practical asymptotic stabilizability of the set by sampled-data state feedback that is discontinuous, in general. Because sensors add cost to a control system, it is natural to ask: under what additional conditions can this property be achieved using output feedback? For this purpose, we used the notion of fast observability by a finite dimensional observer when driven by a "universal input". We showed that this notion of observability is enough to generalize the state feedback result to the output feedback case. This type of observability is much weaker than what had been used previously to achieve stabilization by output feedback. Moreover, the previous results did not permit results based on discontinuous state feedback. The control method that we employed uses a small, initial piece of each sampling period to construct an estimate of the state, via a high-gain observer. During this observation period, an input that guarantees observability is applied. During the rest of the sampling period, the control applied is the state feedback function evaluated at the value of the state estimate provided by the observer at the end of the observation period. Using this approach, semiglobal, practical asymptotic stability can be established.

#### Nonsmooth optimization

*General Theory:* We used our results on the existence of smooth Lyapunov functions for differential inclusions having asymptotically stable sets to derive a rich class of quasi-Newton algorithms for minimization (or maximization) of locally Lipschitz, regular functions that are not necessarily continuously differentiable and/or convex. (Locally Lipschitz regular functions include convex functions and smooth functions as special cases.) The route to these results starts by showing that, under appropriate technical

assumptions, given a locally Lipschitz, regular function, the trajectories of the differential inclusion arising from multiplying the function's set-valued generalized gradient by a set of positive definite matrices converge to the function's set of stationary points (assumed to be compact), at least when the sublevel sets of the function are compact. Then new results on the existence of Lyapunov functions for these types of differential inclusions imply the existence of a function that, from each point, decreases along every direction in the right-hand side of the differential inclusion. Using this observation, it follows that the Euler approximation of the differential inclusion can serve as the backbone for nonsmooth optimization algorithms. These results have the potential to produce a tremendous impact on computational problems associated with control design, as well as many other problems in engineering design. We used these ideas for nonsmooth optimization problems when no generalized gradient information is available. The perils of attempting to approximate elements of the generalized gradient by finite differences are well-documented. Nevertheless, we were able to show that a system-theoretic point of view and an appropriate generalization of the notion of persistency of excitation from the adaptive control and identification literature produce algorithms for nonsmooth optimization based on (single) finite differences without explicitly requiring generalized gradient information.

*Extremum seeking by nonlinear programming:* A classical control engineering problem is that of extremum seeking in nonlinear dynamical systems. This is the problem of adjusting, on-line, the parameters of a (stable) dynamical system in order to optimize the steady-state value of an output variable. The mapping from parameters to steady-state output values is sometimes called the readout map. Some of the early methods for accomplishing this task relied on probing the system with sinusoidal inputs and correlating the output with the drive signals. This method was well-suited for implementation in the analog computers of the time. Recently, rigorous convergence proofs of these algorithms have been given by other authors.

An alternative, more straightforward mind-frame for solving the extremum seeking problem is to rely on the highly developed tools of nonlinear programming and the ability to implement control algorithms with digital computers. The nonlinear programming algorithms that must be used are ones that only use measurements of the readout map. Of course, measurements of the readout map can often be used to approximate the gradient of the readout map if it exists. If the nonlinear programming algorithm used is robust to small measurement errors and if one waits long enough after issuing new parameter values before measuring the readout map, it is obvious that this approach to extremum seeking will achieve its goal. In an effort to speed up the extremum seeking process, one can take measurements of the output function before the transients have decayed. In this case, the dynamics of the nonlinear programming algorithm interact with the dynamics of the process being optimized, and the convergence proof becomes much more involved. We carried out this analysis, in a very general setting that admits dynamical systems governed by infinite dimensional evolution equations, and have provided guidelines for length of waiting time to optimize the extremum seeking procedure when using the nonlinear programming approach. We applied these results to engine calibration problems on test cells at Ford Research Laboratory.

## **Personnel Supported**

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## **Transitions**

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*Theory transitioned:* Exremum seeking in dynamical systems

*Application:* Rapid engine calibration for a variable cam timing engine.

## **Honors/Awards during period**

2002 IEEE Fellow

AACC 2001 O. Hugo Schuck Best Paper Award

2000 IEEE CDC Student Best Paper Award (advisor to Christopher Kellett)

AACC 1999 Donald P. Eckman Award